

# RS model with the small curvature and Bhabha scattering at the ILC

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## Abstract

The Randall-Sundrum (RS) model with the small curvature is studied. In such a scheme the mass spectrum of Kaluza-Klein (KK) gravitons is similar to that in a model with one extra flat dimension. The gravity effects in the Bhabha scattering at the energy 1 TeV are estimated. The calculations are based on the analytical formula which describes virtual graviton contributions. It takes into account both a discrete character of the mass spectrum and nonzero widths of the KK gravitons.

## 1 RS model with the small curvature

The Randall-Sundrum (RS) model [1] is realized in a slice of the  $\text{AdS}_5$  space-time with the background warped metric:

$$ds^2 = e^{2\kappa(\pi r - |y|)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

where  $y = r\theta$  ( $-\pi \leq \theta \leq \pi$ ),  $r$  being the “radius” of extra dimension, and  $\eta_{\mu\nu}$  is the Minkowski metric. The points  $(x_\mu, y)$  and  $(x_\mu, -y)$  are identified.

The parameter  $\kappa$  defines a 5-dimensional scalar curvature of the  $\text{AdS}_5$  space. Namely, the Ricci curvature invariant  $\mathcal{R}^{(5)} = -20 \kappa^2$ . That is why, in what follows we will call  $\kappa$  “curvature”.

There are two 3D branes in the model with equal and opposite tensions located at the point  $y = \pi r$  (called the *TeV brane*) and point  $y = 0$

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(referred to as the *Plank brane*). If  $k > 0$ , the tension on the TeV brane is negative, whereas the tension on the Planck brane is positive. All the SM fields are confined to the TeV brane, while the gravity propagates in all five dimensions.

It is necessary to note that the metric (1) is chosen in such a way that 4-dimensional coordinates  $x_\mu$  are Galilean on the TeV brane where all the SM field live, since the warp factor is equal to unity at  $y = \pi r$ .<sup>1</sup>

By integrating a 5-dimensional action over variable  $y$ , one gets an effective 4-dimensional action, that results in the “hierarchy relation” between the reduced Planck scale  $\bar{M}_{\text{Pl}}$  and 5-dimensional gravity scale  $\bar{M}_5$ :

$$\bar{M}_{\text{Pl}}^2 = \frac{\bar{M}_5^3}{\kappa} (e^{2\pi\kappa r} - 1) . \quad (2)$$

The reduced 5-dimensional Planck mass  $\bar{M}_5$  is related to the Planck mass by  $M_5 = (2\pi)^{1/3} \bar{M}_5 \simeq 1.84 \bar{M}_5$ .

From the point of view of a 4-dimensional observer located on the TeV brane, there exists an infinite number of graviton KK excitations with masses

$$m_n = x_n \kappa, \quad n = 1, 2 \dots , \quad (3)$$

where  $x_n$  are zeros of the Bessel function  $J_1(x)$ .

The interaction Lagrangian on the visible brane looks like the following:

$$\mathcal{L} = -\frac{1}{\bar{M}_{\text{Pl}}} T^{\mu\nu} G_{\mu\nu}^{(0)} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^{\infty} G_{\mu\nu}^{(n)} + \frac{1}{\sqrt{3}\Lambda_\pi} T^\mu_\mu \phi . \quad (4)$$

Here  $T^{\mu\nu}$  is the energy-momentum tensor of the matter on this brane,  $G_{\mu\nu}^{(n)}$  is a graviton field with the KK-number  $n$ , and  $\phi$  is a scalar field called radion. The parameter  $\Lambda_\pi$  in Eq. (4),

$$\Lambda_\pi = \bar{M}_5 \left( \frac{\bar{M}_5}{\kappa} \right)^{1/2} , \quad (5)$$

is a physical scale on the TeV brane.

In most of the papers which treat the RS model,<sup>2</sup> the hierarchy relation is of the form

$$\bar{M}_{\text{Pl}}^2 = \frac{\bar{M}_5^3}{\kappa} (1 - e^{-2\pi\kappa r}) , \quad (6)$$

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<sup>1</sup>To get a right interpretation, one has to calculate the masses on each brane in the Galilean coordinates with the metric  $g_{\mu\nu} = (-1, 1, 1, 1)$  (for details, see Ref. [2]).

<sup>2</sup>Including the original one [1].

while the curvature and 5-dimensional gravity scale are chosen to be of the order of the Planck mass:

$$\kappa \sim \bar{M}_5 \sim \bar{M}_{\text{Pl}} . \quad (7)$$

In such a scheme, one obtains a series of massive graviton resonances in the TeV region which interact rather strongly with the SM fields, since  $\Lambda_\pi \sim 1$  TeV on the TeV brane. The lightest modes of the KK graviton have masses around 1 TeV. Thus, an experimental signature of the “*large curvature option*” of the RS model is the real or virtual production of the massive KK graviton resonances.

There are lower bounds on the mass of the lightest KK mode, coming from the measurements of  $e^+e^-$  and  $\gamma\gamma$  final states on the Tevatron [4]:

$$\begin{aligned} m_1 &> 875 \text{ MeV} \quad (\text{CDF}) , \\ m_1 &> 865 \text{ MeV} \quad (\text{D0}) . \end{aligned} \quad (8)$$

The LHC search limit on  $m_1$  is [5]

$$m_1 \sim 0.7 - 0.8 \text{ TeV} \quad (9)$$

for the di-jet mode and integrated luminosity  $\mathcal{L} = 0.1 \text{ fb}^{-1}$ . For the di-photon mode and  $\mathcal{L} = 10 \text{ fb}^{-1}$ , the limit looks like [5]

$$m_1 \sim 1.31 - 3.47 \text{ TeV} . \quad (10)$$

In the case of large curvature (7), the size of the  $AdS_5$  slice is extremely small:

$$r \simeq 60 l_{\text{Pl}} , \quad (11)$$

where  $l_{\text{Pl}}$  is a Planck length. Thus, in order to explain the huge value of  $\bar{M}_{\text{Pl}}$ , one has to introduce large mass scales  $\bar{M}_5$ ,  $\kappa$ , as well as  $1/r$ . In other words, *hierarchy problem is not solved, but reformulated* in terms of the new parameter related to the size of the bulk along the extra dimension.<sup>3</sup>

In the present paper we will consider the “*small curvature option*” of the RS model [6, 7]. In what follows, the 5-dimensional reduced Planck mass  $\bar{M}_5$  is taken to be one or few TeV. Following Ref. [6]-[8], we chose the parameter  $\kappa$  to be 900 MeV – 1 GeV. These values of  $\kappa$  obey the bounds derived in Ref. [6]:

$$10^{-5} \leq \frac{\kappa}{\bar{M}_5} \leq 0.1 . \quad (12)$$

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<sup>3</sup>A similar shortcoming exists in models with large extra dimensions.

Then the mass of the lightest KK excitation  $m_1 \simeq 3.4 - 3.8$  GeV.

On the contrary, the value of the mass scale  $\Lambda_\pi$  (5) is rather large:

$$\Lambda_\pi = 100 \left( \frac{M_5}{\text{TeV}} \right)^{3/2} \left( \frac{100 \text{ MeV}}{\kappa} \right)^{1/2} \text{TeV} . \quad (13)$$

It immediately follows from Eqs. (4), (13) that there is no problem with the radion field  $\phi$  in our scheme, since its coupling to the SM fields ( $\sim 1/\sqrt{3}\Lambda_\pi$ ) is strongly suppressed.

As for the massive gravitons, their couplings are also defined by the scale  $\Lambda_\pi$  (13). However, the smallness of the coupling is compensated by the large number of the gravitons that can be produced in any inclusive process. As a result, magnitudes of cross sections will be defined by the 5-dimensional gravity scale  $\bar{M}_5$  and collision energy  $\sqrt{s}$  (see below).

Thus, we have an infinite number of low-mass KK resonances with the small mass splitting, in contrast with the standard RS scenario (7).<sup>4</sup> Nevertheless, due to the warp geometry of the  $AdS_5$  space-time, the RS model with the small curvature differs significantly from the ADD model [9] (at least, at  $\kappa \gg 10^{-22}$  eV), as was demonstrated in Ref. [7].

Let us stress that the hierarchy relation (2) explains the value of the 4-dimensional Planck mass without introducing large mass scales, if one put  $r = 9.7/\kappa \simeq 10 \text{ GeV}^{-1}$ .

Recently, the small curvature option of the RS model has been checked by the DELPHI Collaboration. The gravity effects were searched for by studying photon energy spectrum in the process  $e^+e^- \rightarrow \gamma + \cancel{E}_\perp$ . No deviations from the SM prediction were seen. As a result, the following bound has been obtained [11]:

$$M_5 > 1.69 \text{ TeV} \pm 3\% , \quad (14)$$

that corresponds to the reduced 5-dimensional scale  $\bar{M}_5 > 0.92 \text{ TeV}$  (see the relation between  $M_5$  and  $\bar{M}_5$  after Eq. (2)).

Note that this limit could not be inferred from the limits already given for larger than two flat dimensions owing to the totally different spectrum of the photon [11].

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<sup>4</sup>Bounds (8)-(10) *are not valid* for the small curvature scenario of the RS model.

## 2 Virtual KK gravitons in the Bhabha scattering

Following Ref. [14], we will consider now the Bhabha process at the ILC collider [12] mediated by massive graviton exchanges:<sup>5</sup>

$$e^+ e^- \rightarrow G^{(n)} \rightarrow e^+ e^- . \quad (15)$$

The collision energy  $\sqrt{s}$  is taken to be 1 TeV. It means that we are working in the following region:

$$\Lambda_\pi \gg \sqrt{s} \sim M_5 \gg \kappa . \quad (16)$$

The matrix element of the process (15) looks like

$$\mathcal{M} = \mathcal{A} \mathcal{S} . \quad (17)$$

The first factor in Eq. (17) is the contraction of the tensor part of the graviton propagator,  $P^{\mu\nu\alpha\beta}$ , and the energy-momentum tensor of the electron,  $T_{\mu\nu}^e$ :

$$\mathcal{A} = T_{\mu\nu}^e P^{\mu\nu\alpha\beta} T_{\alpha\beta}^e . \quad (18)$$

The second factor in Eq. (17) is *universal* for all types of processes mediated by  $s$  or  $t$ -channel exchange of the massive KK excitations. Let us consider first the contribution to  $\mathcal{S}$  from the  $s$ -channel gravitons:

$$\mathcal{S}(s) = \frac{1}{\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{s - m_n^2 + i m_n \Gamma_n} . \quad (19)$$

Here  $\Gamma_n$  denotes the total width of the graviton with the KK number  $n$  and mass  $m_n$  [13]:

$$\frac{\Gamma_n}{m_n} = \eta \left( \frac{m_n}{\Lambda_\pi} \right)^2 , \quad (20)$$

where  $\eta \simeq 0.09$ .

Note that the main contribution to sum (19) comes from the region  $n \sim \sqrt{s}/\kappa \gg 1$ . For such large values of the KK number  $n$ , *nonzero widths of the gravitons should be taken into account*.

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<sup>5</sup>The processes  $e^+ e^- \rightarrow \mu^+ \mu^-$ ,  $e^+ e^- \rightarrow \gamma \gamma$  are also promising reactions.

The sum in Eq. (19) can be calculated analytically by the use of the formula [15]

$$\sum_{n=1}^{\infty} \frac{1}{z_{n,\nu}^2 - z^2} = \frac{1}{2z} \frac{J_{\nu+1}(z)}{J_{\nu}(z)} , \quad (21)$$

where  $z_{n,\nu}$  ( $n = 1, 2 \dots$ ) are zeros of the function  $z^{-\nu} J_{\nu}(z)$ . As a result, the following explicit expression can be obtained from (19) (for details, see Ref. [7]):

$$\mathcal{S}(s) = -\frac{1}{4\bar{M}_5^3 \sqrt{s}} \frac{\sin 2A + i \sinh 2\varepsilon}{\cos^2 A + \sinh^2 \varepsilon} , \quad (22)$$

with the notations

$$A = \frac{\sqrt{s}}{\kappa} + \frac{\pi}{4}, \quad \varepsilon = \frac{\eta}{2} \left( \frac{\sqrt{s}}{\bar{M}_5} \right)^3 . \quad (23)$$

As one can see from (22), the magnitude of  $\mathcal{S}(s)$  is defined by  $\bar{M}_5$  and  $\sqrt{s}$ , *not by*  $\Lambda_{\pi}$ , although the latter describes the graviton coupling with the matter in the effective Lagrangian (4).

The following inequalities result from (22):

$$-\coth \varepsilon \leq \text{Im } \tilde{\mathcal{S}}(s) \leq -\tanh \varepsilon \quad (24)$$

$$|\text{Re } \tilde{\mathcal{S}}(s)| \leq \frac{1}{1 + 2 \sinh^2 \varepsilon} , \quad (25)$$

$$\left| \frac{\text{Re } \tilde{\mathcal{S}}(s)}{\text{Im } \tilde{\mathcal{S}}(s)} \right| \leq \frac{1}{\sinh 2\varepsilon} , \quad (26)$$

where the notation  $\tilde{\mathcal{S}}(s) = [2\bar{M}_5^3 \sqrt{s}] \mathcal{S}(s)$  was introduced. Note that the ratio  $|\text{Re } \mathcal{S}(s)/\text{Im } \mathcal{S}(s)|$  decreases with energy and becomes small at  $\sqrt{s} \simeq 3\bar{M}_5$ , while  $\text{Im } \mathcal{S}(s)$  tends to  $(-1)/(2\bar{M}_5^3 \sqrt{s})$ .

Should one ignores the widths of the massive gravitons, and replace a summation in KK number (19) by integration over graviton masses,

$$dn = \frac{\bar{M}_{\text{Pl}}^2}{2\pi \bar{M}_5^3} dm , \quad (27)$$

he gets [7, 8]:

$$\text{Im } \mathcal{S}(s) = -\frac{1}{2\bar{M}_5^3 \sqrt{s}} , \quad \text{Re } \mathcal{S}(s) = 0 , \quad (28)$$

in contrast to the exact formula (22).

However, the series of low-massive resonances in the RS model with the small curvature can be replaced by the continuous spectrum *only in a trans-Planckian energy region*  $\sqrt{s} \gtrsim 3\bar{M}_5$ . It can be understood as follows. One may regard the set of narrow graviton resonances to be the continuous mass spectrum (within a relevant interval of  $n$ ), if only

$$\Delta m_{\text{KK}} < \Gamma_n \quad (29)$$

is satisfied, where  $\Delta m_{\text{KK}}$  is the mass splitting. As was shown in Ref. [7], the inequality (29) is equivalent to the above mentioned inequality

$$\sqrt{s} \gtrsim 3\bar{M}_5. \quad (30)$$

We are working in another kinematical region,  $\sqrt{s} \lesssim 3\bar{M}_5$ , since the 5-dimensional scale  $\bar{M}_5$  is assumed to be equal to (or larger than) 1 TeV, while the collision energy is fixed to be 1 TeV. That is why, for our calculations we will use the analytical expression (22) which takes into account both the discrete character of the graviton spectrum and finite widths of the KK gravitons.

Our main formula (22) can be also applied to the scattering of the brane particles, induced by exchanges of  $t$ -channel gravitons [6, 13]. In the kinematical region  $\bar{M}_5^3/\kappa \gg -t \gg \kappa^2$ , the contribution from the  $t$ -channel gravitons looks like [7]:

$$\mathcal{S}(t) = -\frac{1}{2\bar{M}_5^3\sqrt{-t}}. \quad (31)$$

Let us now apply our theoretical formulae (22), (31) for estimating virtual graviton contributions to the process (15). The relations of cross sections with the quantities  $\mathcal{S}(s)$  and  $\mathcal{S}(t)$  can be found in Ref. [8].

Note first that at the LEP2 energy ( $\sqrt{s} = 200$  GeV) the gravity effects are very small with respect to the SM cross section [8, 14]. Thus, the above mentioned bound on  $\bar{M}_5$  from LEP (14) should be also applied to the 5-dimensional scale in our scheme.

In Fig. 1 the graviton contribution is presented as a function of the scattering angle of the final electrons at the ILC energy  $\sqrt{s} = 1$  TeV (solid line). Another prediction is also shown (dashed line) which was calculated under the assumption that the dense spectrum of the KK gravitons can be approximated by a continuum [14].

The next figure shows the dependence of the cross section ratios on the gravity scale  $M_5$ . Figs. 3, 4 demonstrate us that these ratios can be very sensitive to small variations of the curvature  $\kappa$  within a narrow region (0.9 GeV – 1 GeV, in our case).

Let us note that the interference of gravity with the SM forces is constructive in the Bhabha scattering, and the ratio  $\sigma(\text{SM} + \text{grav})/\sigma(\text{SM})$  is larger than 1.

Up to now, we considered the fixed collision energy. However, the effects related with non-zero graviton widths remain significant after energy smearing within some interval around average value of  $\sqrt{s}$ , as one can see in Fig. 5. Note that the energy smearing has a small influence on the cross section ratios in the kinematical region  $\cos \theta < -0.8$  [14].

### 3 Conclusions

In the present paper the contributions of the virtual  $s$ - and  $t$ -channel KK gravitons to the Bhabha scattering at the ILC energy  $\sqrt{s} = 1$  TeV were estimated. We have considered the small curvature option of the RS model with two branes ( $\kappa \ll M_5$ ). In such a scheme, the KK graviton spectrum is a series of the narrow low-mass resonances.

The 5-dimensional Planck scale  $M_5$  was taken to be 1.8 TeV and 2.5 TeV, with the curvature parameter being restricted to the small region  $\kappa = 900 \text{ MeV} - 1 \text{ GeV}$ , that means  $\sqrt{s} \sim M_5 \gg \kappa$ . For numerical estimates, we used formula (22), which describes the gravity contribution to the process-independent part of the scattering amplitude  $\mathcal{S}$  (17).

For comparison, we have made calculations by using Eqs. (28) which treat the spectrum of the KK graviton as a continuum. It was demonstrated that the sum in the KK number can not be approximated by integration over graviton mass. Moreover, the graviton widths should be properly taken into account, as formula (22) does. Our predictions are presented in Figs. 1-5 [14]. Everywhere  $\sigma(\text{SM})$  means the SM differential cross section with respect to  $\cos \theta$ , while  $\sigma(\text{SM} + \text{grav})$  means the differential cross section which takes into account both the SM and gravity interactions.

One should conclude that at fixed value of the fundamental gravity scale  $M_5$ , the cross section ratio is very sensitive even to slight variations of the curvature  $\kappa$  (compare Figs. 2, 3 and 4). The ratio  $\sigma(\text{SM} + \text{grav})/\sigma(\text{SM})$  can reach tens at some values of the parameters (see Figs. 4).



Such a behavior of the cross section ratios comes from the explicit form of the function  $\mathcal{S}(s)$  (22). Indeed, at  $\eta(\sqrt{s}/\bar{M}_5)^3 \ll 1$  the parameter  $\varepsilon$  in Eq. (22) is much less than 1, and the function  $\mathcal{S}(s)$  has a significant real part.<sup>6</sup> The interference with the SM contributions results in a nontrivial dependence of  $\sigma(\text{SM} + \text{grav})/\sigma(\text{SM})$  on  $\cos\theta$ . Given the approximation with zero real part is used (28), no interference terms exist, and  $\cos\theta$ -dependence of the cross section ratio becomes similar for all sets of the parameters (dashed curves in Figs. 1-4).

If the parameter  $A = \sqrt{s}/\kappa + \pi/4$  in formula (22) obeys the equation  $\cos A = 0$ , the denominator in (22) becomes small, and, correspondingly, the function  $\mathcal{S}(s)$  becomes large. It results in the rapid variation of the gravity contribution near corresponding values of  $\kappa$ .

It is worth to note that both a discrete character of the mass spectrum and nonzero widths of the KK gravitons are also important in a model with a *flat* compact extra dimension [14].

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<sup>6</sup>Remember that  $\eta(\sqrt{s}/\bar{M}_5)^3 > 1$  in the trans-Planckian region  $\sqrt{s} > 3\bar{M}_5$ .

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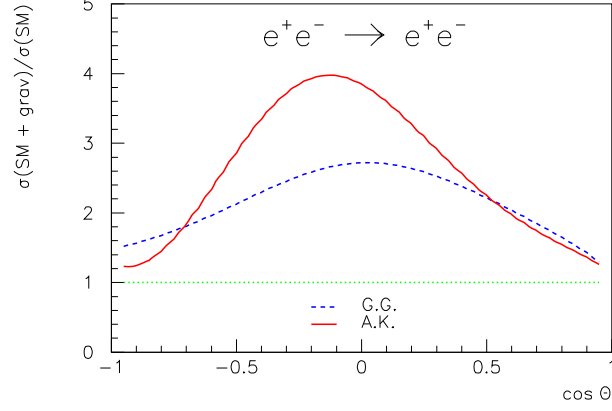


Figure 1: The correction to the Bhabha cross section resulting from virtual graviton exchanges as a function of the scattering angle at the ILC energy  $\sqrt{s} = 1$  TeV. The solid line is our prediction which takes into account both a discrete character of the spectrum and widths of the KK gravitons. The dashed line corresponds to the continuous mass approximation for the graviton spectrum. The parameters are  $M_5 = 2.5$  TeV,  $\kappa = 1$  GeV.

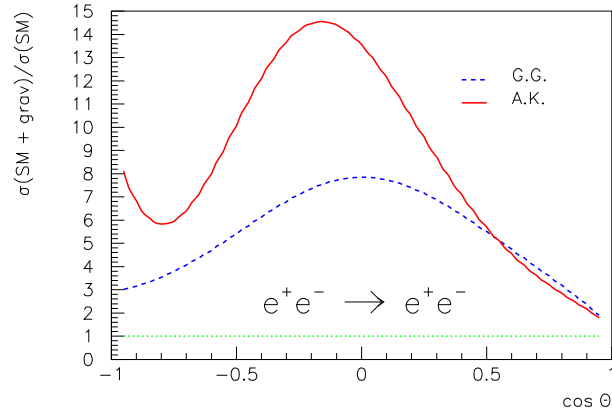


Figure 2: The same as in Fig. 1, with the parameters  $M_5 = 1.8$  TeV,  $\kappa = 1$  GeV.

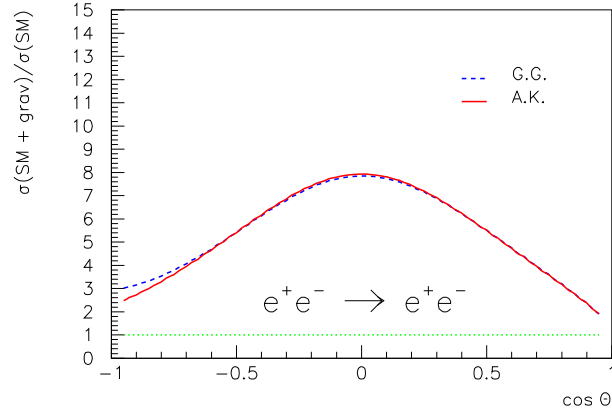


Figure 3: The same energy as in Fig. 2, except for the parameter  $\kappa = 900$  MeV.

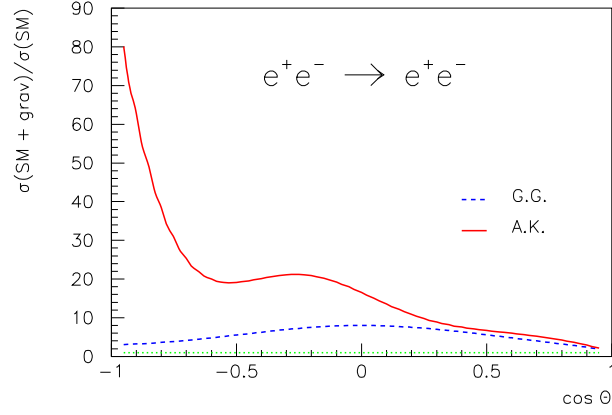


Figure 4: The same energy as in Fig. 2, except for the parameter  $\kappa = 994$  MeV.

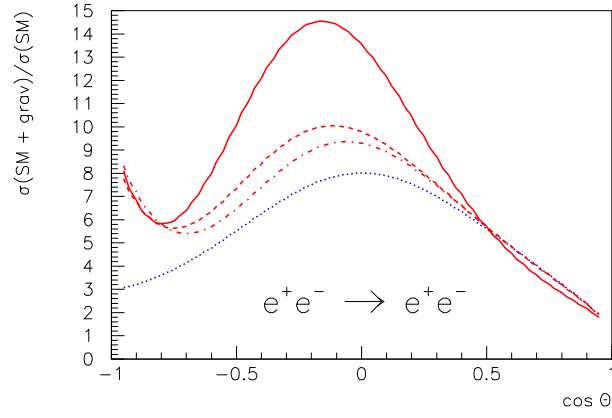


Figure 5: The dashed and dash-dotted lines correspond to smearing of gravity cross section in the interval  $(\sqrt{s}-\Delta\sqrt{s}, \sqrt{s}+\Delta\sqrt{s})$ , with  $\Delta\sqrt{s} = 10$  GeV and  $\Delta\sqrt{s} = 50$  GeV, respectively, while the solid line corresponds to a case without energy smearing. The average energy  $\sqrt{s} = 1$  TeV, the parameters are the same as in Fig. 2. The dotted line is obtained in zero width approximation. The number of resonances which lie within the energy resolution is equal to 6 (31) for  $\Delta\sqrt{s} = 10$  GeV (50 GeV).